Notes on Krugman (1998 BPEA), "It's Baaack!"

Integral

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1 Introduction

Krugman (1998) is a paper about liquidity traps. Ostensibly it's a paper about how liquidity traps happen even in fully-optimizing modern models, but we'll see that it's really just IS-LM in new clothing. The story goes something like this.

1. The Fed's basic instrument is the money supply.
2. If the Fed wishes to increase output, it can simply increase the money supply, which puts downward pressure on interest rates, which boosts output.
3. But there's a zero bound on interest rates: if the money supply gets large enough, then interest rates fall to zero, and any further increase in the money supply won't affect output.
4. So occasionally the central bank might like to stimulate the economy, but cannot, since it can't force interest rates below zero.

so what do we do then? That's the question this paper tries to answer.

The paper includes some lovely sentences.

- "The traditional view that monetary policy is ineffective in a liquidity trap, and that fiscal expansion is the only way out, must therefore be qualified; monetary policy will in fact be effective if the central bank can *credibly promise to be irresponsible,* to seek a higher future price level." (p.139)
- "A liquidity trap involves a kind of credibility problem... if monetary expansion does not work, if there is a liquidity trap, it must be because the public does not expect it to be sustained." (p142)
- "To put it yet another way, closer to the language of applied macroeconomics, if people have low expectations about their future incomes, even with a zero interest rate they may want to save more than the economy can absorb." (p150)

These sentences start a long and fruitful line of thought, roughly stated that "expectations of future policy matter" and "forward guidance matters." We can see these statements hardened by models in Eggertsson and Woodford (2003); these statements are also used to shed light on historical experiences in Eggertsson (2008).

2 A Very Simple Cash-In-Advance Model

This is Krugman, pages 141-146.

2.1 Full Model (Macro 601)

Krugman sets up a cash-in-advance model. For now, suppose that consumers maximize the following utility function:

$$U = c_1^{1-\sigma} + \beta c_2^{1-\sigma} + \beta^2 c_3^{1-\sigma} + \dots$$

where the subscript indicates time, and consumers are subject to a cash-in-advance constraint,

$$P_t c_t \leq M_t.$$

There's no capital, no government spending, and no foreign sector, so $c_t = y_t$ in every period. The central bank picks M_t each period. A *solution* is a sequence of triples $(P_t, i_t, y_t)_{t=1}^{\infty}$ which satisfies:

$$P_t y_t \leq M_t \tag{1}$$

$$\beta(1+i_t)\left(\frac{y_{t+1}}{y_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}} = 1 \tag{2}$$

for $(M_t)_{t=1}^{\infty}$ given. The two equilibrium conditions are the cash-in-advance constraint and the Euler equation, respectively. (The Euler equation maps out consumption bundles that actually maximize the utility function.) We obviously aren't done yet: we have two sequences of equations and three sequences of unknowns. We need one more equation to nail everything down.

2.2 The Simplified Model (Macro 201)

Let us make a bunch of simplifying assumptions. Let us suppose that for $t = 2$ onwards, $P = P^*$ and $y = y^*$. So in equation (2), replace $P_{t+1} = P^*$ and $y_{t+1} = y^*$. Now we only have to solve for three variables (y_1, P_1, i_1) instead of three sequences of variables. Again, we still can't solve for all three, since we only have two equations. So what to do? We'll do two variations: fixing y_1 and solving for (P_1, i_1) or alternatively fixing P_1 and solving for (y_1, i_1).

The simplified model is:

$$P_1 y_1 \leq M_1 \qquad \text{(CIA; LM)}$$

$$1 + i_1 = \frac{1}{\beta}\left(\frac{y^*}{y_1}\right)^{\sigma}\frac{P^*}{P_1} \qquad \text{(Euler; IS)}$$

We're going to use these two equations over and over again. This is just IS-LM. So where are we now?

- By fixing all future variables to their starred values, we turn the whole thing into a two-period problem.
- By fixing all future variables, we condense the problem into one of two equations in three unknowns. By fixing one of today's three variables, the problem becomes one of two equations in two unknowns.
- We're going to two cases. First, assume output is exogenous and try to pin down P_1 and i_1 (the "classical" version of the model). Second, assume that the price level is exogenous and try to pin down y_1 and i_1 (the "Keynesian" version of the model).

So let's do those two cases.

3 Flexible-Price Results

For this section, we're going to fix $y_1 = \bar{y}$ as exogenous and solve for (P_1, i_1). The point is to figure out whether the central bank can freely choose the price level through its choice of the money supply. You can ignore this section if you want, it doesn't really matter for the rest of the paper. This is Krugman, pages 146-148.

3.1 Model Solution with $i > 0$

This is the standard solution. If the CIA constraint binds, then $P_1 = M_1/\bar{y}$, and we are done. This condition holds if and only if i_1 is strictly positive. So one solution is:

$$P_1 = M_1/\bar{y} \tag{LM}$$

$$(1 + i_1) = \frac{1}{\beta}\left(\frac{y^*}{\bar{y}}\right)^{\sigma}\frac{P^*}{P_1}. \tag{IS}$$

Result: the price level today is pinned down by the quantity of money and by the level of output; for a given level of output the central bank can choose P_1 by choosing M_1. The interest rate is whatever it needs to be to make the Euler equation hold.

3.2 Model Solution with $i = 0$

If the CIA constraint does not bind, we have a different solution. In this solution $i_1 = 0$ and we cannot pin down P_1 via the CIA constraint. Instead, we have:

$$i_1 = 0$$

$$P_1 = \frac{1}{\beta}\left(\frac{y^*}{\bar{y}}\right)^{\sigma} P^*$$

so that P_1 is no longer dependent on the money supply! What's going on here? There *is* a quantity M_1 floating around, but today's price level is the minimum of $M_1/\bar{y}$ and $(y^*/\bar{y})^{\sigma}P^*/\beta$. The latter is the relevant constraint.

Try it this way. Suppose you were a monetarist. Then you'd write down the LM curve as:

$$Py = Mv(i)$$

which is the familiar Quantity Theory. The above results show that the velocity of money $v(i)$ is unity when $i > 0$, but the velocity of money is zero when $i = 0$.

Result: thus there exists the possibility of a liquidity trap (failure of the QTM; inability to pin down the price level; unstoppable decline in the current price level) even in a purely flex-price environment.

3.3 Intuition

We normally think the Fed always has control of the price level. This section shows that there can be times where the Fed fails to control the price level. Specifically, when interest rates go to zero, the velocity of money also collapses, and the Fed "loses its grip" on the price level.

In normal times, when the Fed gives you a dollar you try to spend that dollar; since output is fixed by assumption, you end up bidding up the price level by a dollar. But in a liquidity trap, the Fed gives you a dollar and you choose not to spend the dollar. Why not? Because you also want to satisfy your Euler equation, and in a liquidity trap your Euler equation is telling you that your current spending is "just right."

4 Sticky-Price Results

Okay, now do the *exact same exercise* as above, but this time fix $P_1 = \bar{P}$ and solve for (y_1, i_1). We are now even more firmly in IS-LM territory. This is Krugman, pages 148-150.

4.1 Model Solution when $i > 0$

We again have two solutions. First is the "normal" case:

$$y_1 = M_1/\bar{P} \tag{LM}$$

$$1 + i_t = \frac{1}{\beta}\left(\frac{y^*}{y_1}\right)^{\sigma}\frac{P^*}{\bar{P}} \tag{IS}$$

Result: the central bank gets to pick real output. Current GDP is nailed down by the current real money stock. The interest rate is determined via the Euler equation. Everything is fine. If output falls for some reason, the central bank can counteract that fall by increasing M.

4.2 Model Solution when $i = 0$

Second is the "liquidity trap" case. If we increase M too much, then i falls to zero, and we then have:

$$i_1 = 0$$

$$y_1 = \left(\frac{1}{\beta}\frac{P^*}{\bar{P}}\right)^{1/\sigma} y^*$$

where y_1 becomes unhinged from M, and faces an upper bound. The Fed can no longer affect y_1 through M! Current output is nailed down by expected inflation and the expected future output level.

Result: the central bank cannot generate arbitrarily high y by manipulating M. It faces a hard maximum: with interest rates stuck at zero, the Euler equation (IS equation) determines y_1. If the central bank is to affect output at the zero lower bound, it must do so by influencing the IS equation.

Result: the central bank could still increase current GDP if it could commit to a higher future price level. The central bank must commit to being irresponsible; it must commit to higher inflation; it must commit to a higher future price level path.

4.3 Intuition

Similar intuition, but it might even be more intuitive this time, since we're picking output.

In normal times, the Fed gives you a dollar and you spend the dollar. This increases your current consumption. You'd like to also increase future consumption, but the interest rate adjusts so that future consumption is "just right." The velocity of money is unity.

In a liquidity trap, the Fed gives you a dollar. But since interest rates are stuck at zero, the Euler equation tells you that your current consumption is "just right," given expectations of future consumption. So you hoard the dollar instead of spending it. The velocity of money is zero. More money just means cash stuffed under mattresses (or cash held in bank vaults).

Draw an IS-LM diagram. Draw it so that IS cuts zero at some sufficiently high y. Draw it with a normal equilibrium. Now, push LM out so far that the equilibrium implies a negative interest rate. But interest rates can't go below zero! So instead, equilibrium is just $i = 0$, and y is determined by wherever the IS curve cuts zero. That's all Krugman is saying.

5 Scott Sumner version

This is a detour. Suppose $\sigma = 1$. Then the Euler equation becomes:

$$1 + i_t = \frac{1}{\beta}\left(\frac{y^*}{y_1}\right)\frac{P^*}{P_1} \qquad \text{(IS, Sumner version)}$$

And let's rearrange that equation to read:

$$Py = \frac{1}{\beta}\frac{P^* y^*}{1+i}.$$

There's nothing new here: this is just the same IS curve as we've had all along, but rearranged. Now we are in comfortably Scott Sumner territory. Define $N = Py$ and we have:

$$N = \frac{1}{\beta}\frac{N^*}{1+i} \qquad \text{(IS, Sumner version, NGDP style)}$$

so that the Fed can influence NGDP *either* by manipulating the interest rate *or* by manipulating expected future NGDP.

5.1 Model Solution with $i = 0$

Suppose we're at the ZLB. Then,

$$N = \frac{N^*}{\beta}$$

and the implication is clear. Increase expected future NGDP! Who cares about the liquidity trap when you can just boost expected future nominal income? But of course, the assumption is that the Fed can indeed influence expected future NGDP. Note that there is something of an overshooting argument here. The solution to bad shocks is to commit to a higher expected future path of NGDP than you otherwise would have committed to.

5.2 Intuition

The intuition is all coming from the Euler equation. Here's the basic line of thought.

1. People like to smooth their consumption over time. The Euler equation is a mathematical version of that idea.
2. Here's how to interpret the Euler equation in English: people say, "well, given my expected future income and the interest rate, I'm optimizing my current income."
3. So there are two ways to make people want to spend more today: cut interest rates or increase expected future income.
4. Usually the Fed cuts interest rates.
5. But at the ZLB, it can't do that anymore.
6. So, at the ZLB, the Fed needs to increase expected future income if it wants to spur consumption today.

6 Credibility and Monetary Policy

This is Krugman, pages 160-165. He spends a few pages (150-160) doing a few variations: including capital, including a foreign sector, and including fiscal policy. Let's zoom in on the bit about credibility.

6.1 Expected Future NGDP Matters!

Specifically, set $\sigma = 1$ again and hone in on the Euler equation:

$$y_1 = \frac{1}{\beta}\frac{N^*}{P_1} \tag{IS}$$

and focus now now on the N^* term. If the central bank can credibly boost expected future nominal spending, it can engineer an increase in output or prices or both. There, done, no more liquidity trap: a liquidity trap is always and everywhere a commitment problem, an inability to commit to a higher future level of nominal spending.

Note that there is some overshooting here; the expected future NGDP path needs to be higher than before. However, if expected future NGDP collapses, then indeed even restoring expected NGDP to N^* will help in stabilizing output.

In the preceding few paragraphs I've stepped away from Krugman, who focuses on the expected future price level rather than the expected future level of nominal income. Krugman recommends an inflation target of 4%. However, the NGDP version flows naturally from his own model, and we can just as easily talk about a higher price-level target or even a higher NGDP target.

6.2 More Intuition

Krugman shows, fairly convincingly, that simple expansion of the current money supply could fail to increase GDP. Here's a sketch. We have two equations: a cash-in-advance equation and an Euler equation. We're going to fix expected future P and y. We're going to fix current P. Normally, the cash-in-advance constraint binds, and $y = M/P$, so the central bank can perfectly choose output. However, when $i = 0$, the *Euler equation* becomes the constraint: velocity collapses, and monetary policy is "pushing on a string." The only way out is to manipulate the other side of the Euler equation; that is, manipulate expected future P or y.

Let's take it slowly. Why doesn't M/P pin down current output in a liquidity trap situation? The answer is that in a liquidity trap, there is a *tension* between the cash-in-advance constraint and the Euler equation, and the Euler equation wins. You get an extra dollar, but don't spend it, because you're not trying to max $U(Y)$, you're trying to max $U(Y) + \beta U(Y')$. Since Y' is pinned down, you can be *satiated* in output today.

At the same time, the Euler equation shows how increasing expected future NGDP can raise demand today. By raising expected future NGDP, households are motivated to increase current spending for PIH/consumption-smoothing reasons. Again, though, do we really think that's what's going on in a liquidity trap? Maybe. Maybe not.

The punchline: in a liquidity trap, households are *satiated in consumption today*, given their expectations of consumption tomorrow. More cash just means more cash under the mattress. The only way out is to raise expected future nominal spending, and thereby spur higher current nominal spending via PIH/consumption-smoothing motives. That means higher future NGDP, or higher expected inflation, or lower real interest rates, whatever.

References

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